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METHODS FOR ANALYZING CONSTANT-HEAD WELL PERMEAMETER DATA

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Abstract

The constant-head well permeameter has proven to be a useful and versatile instrument for determining the in situ hydraulic properties of soils in the unsaturated (vadose) zone. The flow measurements are obtained under conditions of saturated-unsaturated, three-dimensional flow in the unsaturated zone. As a consequence, the steady-state flow rate out of the permeameter is determined by both the field-saturated hydraulic conductivity (K_{fs}) and the matric flux potential (ϕ_m) of the unsaturated soil. Because both K_{fs} and ϕ_m contribute to the flow, calculation of these parameters from well permeameter data requires either the solution of two (or more) simultaneous equations, or reduction of the problem to one equation in one unknown if additional information is known or estimated. Use of the simultaneous-equations approach in heterogeneous soils can result in a high percentage of invalid (i.e., negative) K_{fs} and ϕ_m values. Negative results can be avoided and good estimates obtained, however, by using an independent measurement or site estimate of the ratio $\alpha^* = K_{fs}/\phi_m$.

METHODS for analyzing constant-head well permeameter data from measurements above the water table and in the unsaturated zone have progressed from analyses based solely on saturated flow and the hydrostatic pressure contribution to analyses that include saturated-unsaturated flow and hydrostatic pressure, gravity, and capillary contributions to flow. We will discuss the major elements of the physics of well permeameter flow and clarify how the three components of flow (i.e., hydrostatic pressure, gravity, and capillarity) can be taken into account when determining the hydraulic parameters from measurements in field soils, which tend to be more or less heterogeneous in nature.

We begin with Glover's approximate analytical solution (Zangar, 1953), which is based on Laplace's equation, and which assumes the soil to be staturated throughout at all times during the measurement. This assumption would only be true if the measurement was carried out beneath the water table. The measurement, however, is carried out in the vadose zone above the water table, where the soil is unsaturated. Steady flow produces a small inner saturated bulb adjacent to the well, encased within a larger outer wetted, but unsaturated, volume (Philip, 1985; Elrick et al., 1989; Stephens and Neuman, 1982). As a consequence, combined saturated-unsaturated flow occurs. Stephens and Neuman (1982) were the first to comprehend the nature of the saturated-unsaturated flow system, and they consequently based their empirical analysis on the Richards' equation for saturated-unsaturated flow, rather than on Laplace's equation. Later, Reynolds et al. (1985) and Philip (1985) developed different, but comparable, approximate analytical solutions of Richards' equation to account for both the saturated and unsaturated components of flow from an auger or bore hole above the water table. It has been found subsequently that all three solutions (i.e., those of Stephens and Neuman, 1982; Reynolds et al., 1985; and Philip, 1985) give comparable results when predicting the steady flow rate out of the well (Elrick et al., 1989, 1990; Heinen and Raats, 1990). The solutions differ slightly, however, in the way in which they partition the flow into its field-saturated and unsaturated components. In the following discussion, we use the solutions given by Reynolds et al. (1985) and Elrick et al. (1989).

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Steady Flow from the Constant-Head Well Permeameter

The soil hydraulic parameters controlling steady flow from a constant-head well permeameter can be written as (Elrick et al., 1989):

$$K_{fs} = CQ/(2\pi H^2 + \pi a^2 C + 2\pi H/\alpha^*) \quad [1]$$

$$\phi_m = CQ/[(2\pi H^2 + \pi a^2 C)\alpha^* + 2\pi H] \quad [2]$$

where K_{fs} (L T^{-1}) is the field-saturated hydraulic conductivity, ϕ_m (L^2 T^{-1}) is the matric flux potential, α^* (L^{-1}) is the ratio of K_{fs}/ϕ_m, Q (L^3 T^{-1}) is the steady intake rate of water, H (L) is the constant height of ponded water in the well, a (L) is the radius of the well, and C is the dimensionless shape factor. In both Eq. [1] and [2], the three terms in the denominator on the right-hand side represent, respectively, the approximate contributions of hydrostatic pressure, gravity, and capillarity to the total flow out of the well.

Glovers Solution. Glover's solution (Zangar, 1953), as stated by Amoozegar (1989) in his Eq. [1a] and [1b], is given by

$$K_{fs} = CQ/(2\pi H^2) \quad [3]$$

where the shape coefficient, C, is given by

$$C = \sinh^{-1}(H/a) - (a^2/H^2 + 1)^{1/2} + a/H \quad [4]$$

The development of Eq. [3] and [4] by Glover (Zangar, 1953) was a truly landmark accomplishment at the time, but it contains some important deficiencies. As mentioned above, Glover's analysis makes the unrealistic assumption that the soil is saturated everywhere (because it is based on Laplace's equation). In addition, the C coefficient in Glover's solution is based on an approximate analytical analysis, which does not satisfy the boundary condition of constant hydraulic head along the wetted surface of the well. Finally, the Glover analysis does not take gravity flow out of the well into account (Reynolds et al., 1983).

Reynolds et al. (1983) and Philip (1985) published improvements on Glover's C value (Fig. 1), which gave better approximations of the boundary conditions along the submerged surface of the well. The "JRP" (Philip, 1985) and "half-source" (Reynolds et al., 1983) C values are based on improved analytical boundary representations of the well, and the "numerical" C values (Reynolds et al., 1983) are derived from a numerical solution of Laplace's equation. These C values are all based on Laplace's equation, and therefore make the unrealistic assumption that only saturated flow occurs. (This is in contrast with the C values reported in Reynolds and Elrick [1987], which are based on Richards' equation and which include capillarity and gravity). It is clear from Fig. 1 that the Glover C values underestimate significantly the JRP, half-source and numerical C values. Reynolds et al. (1983) showed that this underestimate translates into a 40% underestimate of K_{fs} when $H/a = 10$. We show below that the total error in calculating K_{fs} via the Glover solution can be much higher and easily several hundred percent or more, because the Glover solution assumes that $\alpha^* \rightarrow \infty$ (i.e., zero capillarity) and that gravity is negligible, whereas a more appropriate assumption might set $\alpha^* = 12\ \mathrm{m}^{-1}$ and include gravity.

Contrary to the assertion of Amoozegar (1989), the Glover analysis does not account for gravity flow out of the well. The development in Reynolds et al. (1983) shows that the Glover analysis considers only flow due to the hydrostatic pressure of the ponded water in the well. This is readily seen by comparing Eq. [3] with Eq. [1]. In their analysis, Reynolds et al. (1983) adds the gravity component of flow to the Glover solution (their Eq. [11]). They also show that the importance of gravity is inversely proportional to the square of the H/a ratio. Using Eq. [11] of Reynolds et al. (1983) and Fig. 1, it is evident that gravity accounts for only about 1.5% of the total flow out of the well when $H/a = 10$, but >30% of the flow when $H/a = 0.5$.

Rewriting Eq. [1] and [2] as

$$Q = (2\pi H^2/C + \pi a^2)K_{fs} + (2\pi H/C)\phi_m \quad [5]$$

$$= [2\pi H^2/C + \pi a^2 + 2\pi H/(\alpha^* C)]K_{fs} \quad [6]$$

$$= [(2\pi H^2/C + \pi a^2)\alpha^* + 2\pi H/C]\phi_m \quad [7]$$

shows that Q depends on both the saturated (K_{fs}) and unsaturated (ϕ_m) components of hydraulic conductivity. This is best illustrated by reference to Fig. 2, where the comparable integral expressions for saturated and unsaturated flow are given by $\phi_v = K_{fs}H$ and $\phi_m = \int_{\psi_i}^{0} K(\psi)d\psi$, respectively. The background or initial water potentional, ψ_i (L), corresponds to the initial water content in the soil (assumed to be homogeneous) surrounding the well at the start of a measurement. For completeness, the residual potential, ϕ_r

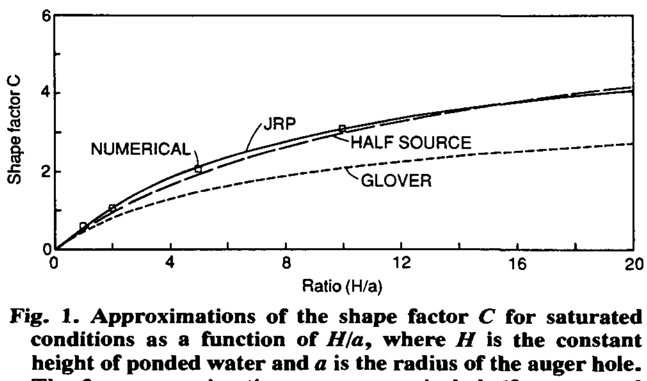

Fig. 1. Approximations of the shape factor C for saturated conditions as a function of H/a, where H is the constant height of ponded water and a is the radius of the auger hole. The four approximations are: numerical, half source and Glover (Reynolds et al., 1983), and JRP (Philip, 1985).

(L^2T^{-1}), is also given as well as the Green and Ampt suction at the wetting front, h_f (L). The residual potential is assumed to be negligible, compared with ϕ_m, for soils at or below field capacity.

Note that Glover's solution is given by the first term only of Eq. [5], [6], and [7]; i.e., $Q = 2\pi H^2 K_{fs}/C$ and therefore completely ignores the contributions of gravity and capillarity. Reynolds et al. (1985) and Philip (1985) have shown that, in dry unstructured clay soils, the capillarity component of flow (i.e., the ϕ_m term) can be many times larger than the saturated component. As a consequence, use of the first term only of Eq. [5], [6], and [7] or Glover's solution (Eq. [3] and [4]) can result in overestimates of K_{fs} of more than an order of magnitude (Reynolds et al., 1985; Reynolds and Elrick, 1987).

Richards-Solution (Simultaneous Equations) and the Single-Height Solution. Our first approach (Reynolds et al., 1985) was to solve for the two unknowns in Eq. [5] (i.e., K_{fs} and ϕ_m) by successively ponding water at two heights, H_1 and H_2, measuring Q_1 and Q_2 at these two heights and then solving the resulting two equations simultaneously (hence the designation, *simultaneous equations*). This approach is based on the assumption that the two overlapping spheroidal soil volumes that determine Q_1 and Q_2 are homogeneous. This is often not true in field soils, however, and heterogeneity can lead to negative values of K_{fs} and ϕ_m (Elrick et al., 1989). To completely remove the possibility of negative numbers, we recommend that two single-height analyses (Elrick et al., 1989) be carried out concurrently with the simultaneous-equations analysis. The single-height analysis, however, requires the independent determination or estimation of an additional parameter, α^*.

Although both K_{fs} and ϕ_m may vary by a factor of 10^5 or more from coarse sandy soils to compacted clays, the ratio of $K_{fs}/\phi_m = \alpha^*$ has generally been found, under field conditions, to range across only one to two orders of magnitude (White and Sully,

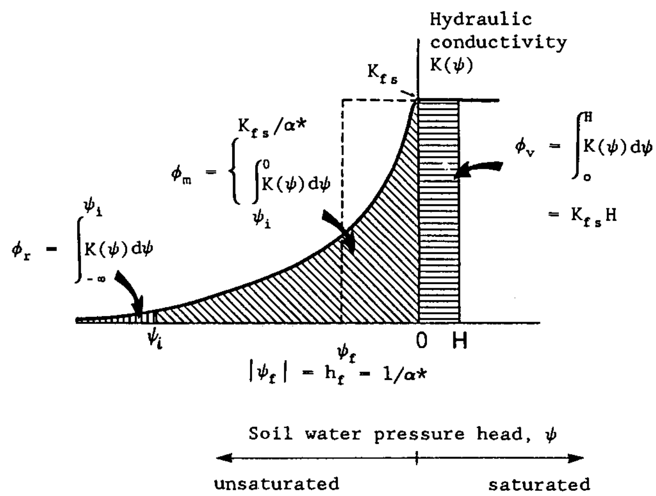

Fig. 2. Hydraulic conductivity, K, as a function of soil water pressure head, ψ, with associated integral properties of residual flux potential ϕ_r, matric flux potential ϕ_m, velocity potential ϕ_v, saturated hydraulic conductivity K_{fs}, the ratio $K_{fs}/\phi_m = \alpha^*$, and the Green and Ampt suction at the wetting front, $h_f = |\psi_f|$.

1987). Estimates of α^* are given in Elrick et al. (1989) and summarized in Table 1.

If a single-height measurement is carried out, our recommendation is that a field evaluation of α^* is superior to simply setting $\alpha^* \rightarrow \infty$, as is the case with Glover's solution. As shown in Eq. [1] and [2], the choice of α^* partitions the flow between the saturated and unsaturated components.

Two (or more) single-height analyses can obviously be averaged, be they Glover-based or otherwise, to obtain an average value of K_{fs}. To apply Glover's solution to a two-height measurement and solve simultaneously, however, as reported by Amoozegar (1989), is mathematically trivial as there is only one unknown, K_{fs}. In addition, Amoozegar's (1989) assumption of unit hydraulic gradient along the wetted surfaces of the well has been shown to be greatly in error (Reynolds et al., 1983; Reynolds, 1986). This assumption is also inconsistent with the non-unit gradient distribution produced by the Glover analysis itself (ref., Reynolds et al., 1983, Fig. 5). Finally, Amoozegar's (1989) Eq. [5a], [5b], and [6] and the subsequent analyses based on these equations are physically incorrect because Q_1 and Q_2 are determined by overlapping spheroidal wetting volumes in unsaturated soil (Philip, 1985; Elrick and Reynolds, 1990), rather than by the nonoverlapping two-layered cylindrical wetting volumes required by Amoozegar's analysis.

Example Calculations

These comments can perhaps be better understood by examining example calculations conducted on a two-height (H_1 = 0.05 m, H_2 = 0.10 m) data set (Table 2). The data chosen are from the Guelph Permeameter operating manual (Soilmoisture Equipment Corporation, 1987, p. 16).

Table 2 compares the Glover, single-height, and simultaneous-equations calculations of K_{fs} and ϕ_m, as well as illustrating the effect of the selection of α^*. It is seen that the Glover calculation (Eq. [3] and [4]) and the one-height calculation (Eq. [1] and [2]) with $\alpha^* \rightarrow \infty$ are essentially equivalent, and that both give an average K_{fs} [i.e., $(K_1 + K_2)/2$] of 6.8×10^{-6} m s^{-1}, which overestimates the simultaneous-equations K_{fs} (K_{SE}) by a factor of about 2.3. As mentioned above, however, the estimate (error) in using the Glover analysis (or $\alpha^* \rightarrow \infty$) can be more than an order of magnitude when the capillarity component of flow is large (Reynolds et al., 1985; Reynolds and Elrick, 1987). A physically more realistic interpretation of the data is obtained by using α^* estimated from a site evaluation of the soil texture and structure. Using α^* = 12 m^{-1}, which is appropriate for the site from which the data set was obtained, $K_{SE} = 2.9 \times 10^{-6}$ m s^{-1} and $(K_1 + K_2)/2 = 3.3 \times 10^{-5}$ ms^{-1}, showing good agreement. The same argument holds true for the matric flux potential calculations, with the average of the two single-height measurements of 2.7×10^{-7} m^2 s^{-1} agreeing well with the simultaneous-equations ϕ_m (ϕ_{SE}) = 3.0×10^{-7} m^2 s^{-1}. The overall agreement of hydraulic conductivity and matric flux potential calculations is good because the α^* calculated from K_{SE}/ϕ_{SE} = 9.7 m^{-1}, which closely approximates the selected value of α^* = 12 m^{-1}. At the other extreme, with $\alpha^* = 0$, the hydrostatic pressure and gravity components of flow are automatically zero and the matric flux potential is overestimated. Our recommendation is that a site evaluation of α^* is superior to using the Glover analysis or to setting $\alpha^* \rightarrow \infty$ when using a single-height analysis.

In Table 3 we show how negative values for K_{SE} and ϕ_{SE} can be obtained and subsequently interpreted. As explained by Elrick et al. (1989), the simultaneous-equations solution assumes that the soil within the two spheroidal measurement volumes is homogeneous, which is seldom the case because of soil horizon development and spatial variability. Measurement errors can also influence the results. Primarily as a result of soil heterogeneity, the ratio of Q_2/Q_1 can be either too high or too low as required by the assumption of homogeneity. If the ratio is too low, the result is a negative value for K_{SE} and an overestimation of ϕ_{SE}, as discussed by Reynolds and Elrick (1985). In Table 3, we arbitrarily chose Q_2^L = 0.75 Q_2, which results in a negative value for K_{SE}. The approach to use here is to realize the limitations of the simultaneous-equations approach and, in these cir-

Table 1. Suggested values for α^* (= field-saturated hydraulic conductivity [K_{fs}]/matric flux potential [ϕ_m]) based on soil structural/textural considerations (for use in Eq. [1] and [2]).

α^*	Comments
m^{-1}	
∞	Essentially the Laplace (Reynolds and Elrick, 1985) or Glover solutions (small differences due to choice of shape factor *C*) with capillarity negligible. Probably applicable to gravels.
36	Coarse sands and highly structured soils.
12	Most structured soils and medium and fine sands.
4	Unstructured fine-textured soils.
1	Compacted clays (e.g., clay liners).
0	The Gardner solution (Reynolds and Elrick, 1985). Pressure and gravity contributions negligible.

Table 2. Illustrative example of hydraulic conductivity and matric flux potential calculations using different techniques. Hydraulic conductivities, K_1 and K_2, and matric flux potentials, ϕ_1 and ϕ_2, are single-height for ponded heights of 0.05 and 0.10 m, respectively, based on Eq. [1] and [2] with the parameter α^* selected from the left-hand column. The values of K_{SE} and ϕ_{SE} are the simultaneous-equations solutions based on Eq. [5]. Steady water-intake rates were $Q_1 = 1.77 \times 10^{-7}$ m^3 s^{-1} and $Q_2 = 2.95 \times 10^{-7}$ m^3 s^{-1} for ponded heights of 0.05 and 0.10 m, respectively, for a well radius of 0.03 m.

α^*	K_1	K_2	$\frac{K_1 + K_2}{2}$	K_{SE}†	ϕ_1	ϕ_2	$\frac{\phi_1 + \phi_2}{2}$	ϕ_{SE}
m^{-1}	10^6 m s^{-1}				10^7 m^2 s^{-1}			
(Glover)	8.1	5.5	6.8	—	—	—	—	—
∞	8.0	5.6	6.8	—	0	0	0	—
36	5.4	4.5	4.9	2.8	1.5	1.2	1.4	3.0
12	3.3	3.2	3.3	2.9	2.8	2.7	2.7	3.0
4	1.5	1.7	1.6	3.0	3.8	4.3	4.0	3.0
1	0.44	0.55	0.50	3.0	4.4	5.4	5.0	3.0
0	0	0	0	—	4.7	6.1	5.4	—

†The small changes in K_{SE} (and ϕ_{SE} in general) are due to the fact that the shape factors for the 0.05- and 0.10-m depths change slightly with α^* (Reynolds and Elrick, 1987, Fig. 6, Curves 2, 3, and 4).

Table 3. Illustration of the effect of heterogeneity and measurement errors on the calculation of field-saturated hydraulic conductivity, K_{SE}, K_1, and K_2, and matric flux potential, ϕ_{SE}, ϕ_1, and ϕ_2. The K_{SE} and ϕ_{SE} values are determined using the simultaneous-equations approach (Eq. [5]) with $H_1 = 0.05$ m, $H_2 = 0.10$ m and $a = 0.03$ m. The K_1 and ϕ_1 values are determined using the single-head calculations (Eq. [1] and [2], respectively) with $\alpha^* = 12$ m^{-1}, $H_1 = 0.05$ m, and $a = 0.03$ m. The K_2 and ϕ_2 values are also determined using Eq. [1] and [2], respectively, but with $\alpha^* = 12$ m^{-1}, $H_2 = 0.10$ m, and $a = 0.03$ m. The steady water-intake rate at 0.05 m, Q_1, remains fixed at 1.77×10^{-7}m^{3}s^{-1}. The effects of relatively small changes in the steady water-intake rate at 0.10 m, Q_2, is illustrated by arbitrarily setting a low value, $Q_2^L = 0.75Q_2$, and a high value, $Q_2^H = 1.5Q_2$, for a well radius of 0.03 m. Note that the underlined values are negatives.

	K_{SE}	$\frac{K_1 + K_2}{2}$	ϕ_{SE}	$\frac{\phi_1 + \phi_2}{2}$
	10^6 m s^{-1}		10^7 m^2 s^{-1}	
$Q_2 = 2.95 \times 10^{-7}$ m^3 s^{-1}	2.9	3.3	3.0	2.7
$Q_2^L = 0.75Q_2$	−0.28	2.9	4.8	2.4
$Q_2^H = 1.5Q_2$	9.1	4.1	−0.57	3.4

cumstances, use the average of the two single-height solutions. In this example, the single-height average K_{fs} of 2.9×10^{-6} m s^{-1} should be chosen as the best estimate, which fortuitously in this example is the same as that obtained by the simultaneous-equations approach using Q_2. In addition, the single-height average ϕ_m of 2.4×10^{-7} m^2 s^{-1} is also a better estimate than $\phi_{SE} = 4.8 \times 10^{-7}$ m^2 s^{-1}. Similar reasoning applies to the interpretation of K_{fs} and ϕ_m when a negative value of ϕ_{SE} is obtained, i.e., use the average K_{fs} and ϕ_m of the single-height calculations. We agree with Amoozegar (1989) that the simultaneous-equations approach works well only in homogeneous material, but we do not agree that Glover's solution is the best alternative for heterogeneous soil conditions.

Thus, when K_{SE} or ϕ_{SE} is either negative or considerably lower than anticipated, the average of the two single-height measurements should be selected as the best estimates for K_{fs} and ϕ_m. It is also important to note that the single-height calculation of K_{fs} and ϕ_m is directly proportional to Q for a fixed α^* (see Eq. [1] and [2]). Thus standard-deviation calculations of K_{fs} and ϕ_m based on multiple measurements at a site using the single-height approach give results that are proportional to the steady-state Q values and are, therefore, representative of the variability of the site. Standard deviations based on the simultaneous-equations calculations generally give larger numbers that are artificially increased because of solving simultaneous equations where K_{fs} and ϕ_m are not precisely constant within the measurement volumes (Elrick et al., 1989).

In many instances, only a single measurement of Q is obtained. If this is the case, use a site evaluation to estimate α^* and thus obtain best estimates of K_{fs} and ϕ_m. For example, if only Q_2 was measured at $H_2 = 0.10$ m and a site evaluation set $\alpha^* = 12$ m^{-1}, the best estimates of K_{fs} and ϕ_m would be 3.2×10^{-6} m s^{-1} and 2.7×10^{-7} m^2 s^{-1}, respectively, as shown in Table 2. Glover's solution overestimates K_{fs} and neglects ϕ_m. In most instances, an improper site evaluation of α^* will be in error at most by only one category, e.g., α^* chosen as 12 m^{-1} whereas 36 m^{-1} would be more appropriate. The result is that K_{fs} and ϕ_m may be in error by a factor of two to three at most. These errors are acceptable, given that K_{fs} ranges from 10^{-9} m s^{-1} for "tight" clays to 10^{-4} m s^{-1} for coarse sands and given the extreme spatial variability of K_{fs} and ϕ_m found in the field (often coefficients of variation of several hundred percent).

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